Applied Mathematics and Computation 308 (2017) 115–129

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A new relaxed PSS preconditioner for nonsymmetric saddle point problems☆

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ARTICLE INFO

MSC:
65F10
65N22

Keywords:
Saddle point problem
Preconditioning
Krylov subspace method
Navier–Stokes equation
GMRES

ABSTRACT

A new relaxed PSS-like iteration scheme for the nonsymmetric saddle point problem is proposed. As a stationary iterative method, the new variant is proved to converge unconditionally. When used for preconditioning, the preconditioner differs from the coefficient matrix only in the upper-right components. The theoretical analysis shows that the preconditioned matrix has a well-clustered eigenvalues around (1, 0) with a reasonable choice of the relaxation parameter. This sound property is desirable in that the related Krylov subspace method can converge much faster, which is validated by numerical examples.

1. Introduction

The steady-state Navier–Stokes system is a basic tool for the modeling of an incompressible Newtonian fluid [30]. Finding effective methods for this system remains a crucial issue for a range of engineering applications [25,26]. Let $\Omega \subset \mathbb{R}^2$ (or $\mathbb{R}^3$) be a bounded, connected domain with a boundary Γ. Given a force field $\mathbf{f}$ and boundary data $\mathbf{g}$, the problem is to ascertain a velocity field $\mathbf{u}$ and a pressure field $\mathbf{p}$ that satisfy the steady-state Navier–Stokes system [15]

$$\begin{aligned} -\nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla\mathbf{p} = \mathbf{f}, &\quad \text{in } \Omega, \\ \operatorname{div}\mathbf{u} = 0, &\quad \text{in } \Omega, \\ \mathbf{u} = \mathbf{g}, &\quad \text{on } \partial\Omega, \end{aligned} \tag{1}$$

where $\nu > 0$ is the kinematic viscosity, Δ is the vector Laplacian, ∇ is the gradient and div is the divergence. The Navier–Stokes Eq. (1) is nonlinear due to the existence of $(\mathbf{u}\cdot\nabla)\mathbf{u}$. Two common strategies, namely Picard iteration and Newton iteration, are available for linearizing (1), which leads to the following Oseen problem

$$\begin{aligned} -\nu\Delta\mathbf{u} + (\omega\cdot\nabla)\mathbf{u} + \nabla\mathbf{p} = \mathbf{f}, &\quad \text{in } \Omega, \\ \operatorname{div}\mathbf{u} = 0, &\quad \text{in } \Omega, \\ \mathbf{u} = \mathbf{g}, &\quad \text{on } \partial\Omega, \end{aligned} \tag{2}$$

☆ This work is supported by National Natural Science Foundation (Nos. 11601323, 11371243), Foundation of Zhejiang Educational Committee (Y201431769) and Young Teacher Training Program of Shanghai Municipality (2015).

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http://dx.doi.org/10.1016/j.amc.2017.03.022
0096-3003/