A new relaxed HSS preconditioner for saddle point problems

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Abstract

We present a preconditioner for saddle point problems. The proposed preconditioner is extracted from a stationary iterative method which is convergent under a mild condition. Some properties of the preconditioner as well as the eigenvalues distribution of the preconditioned matrix are presented. The preconditioned system is solved by a Krylov subspace method like restarted GMRES. Finally, some numerical experiments on test problems arisen from finite element discretization of the Stokes problem are given to show the effectiveness of the preconditioner.

Keywords: Saddle point problems, HSS preconditioner, Preconditioning, Krylov subspace method, GMRES.
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1. Introduction

We study the solution of the system of linear equations with the following block 2×2 structure

$$\mathcal{A}u = \begin{bmatrix} A & B^T \\ -B & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix} \equiv b, \tag{1.1}$$

where $A \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix, $B \in \mathbb{R}^{m\times n}$ with $rank(B) = m < n$. In addition, $x, f \in \mathbb{R}^n$, and $y, g \in \mathbb{R}^m$. We also assume that the matrices A and B are large and sparse. According to Lemma 1.1 in [13] the matrix $\mathcal{A}$ is nonsingular. Such systems are called saddle point problems and appear in a variety of scientific and engineering problems; e.g., computational fluid dynamics, constrained optimization, etc. The readers are referred to [3, 14] for more discussion on this subject.

Several efficient iterative methods have been proposed during the recent decades to solve the saddle point problems (1.1), such as SOR-like method [25], modified block SSOR iteration [1, 2], generalized SOR method [10], Uzawa method [29], parametrized inexact Uzawa methods [11], Hermitian and skew-Hermitian splitting (HSS) iteration methods [4, 8, 7] and so on. However, in some situations, these iterative methods may be less efficient than the Krylov subspace methods [29]. On the other hand, when Krylov subspace methods are applied to the saddle point problem (1.1), tend to converge slowly.

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