Iterative Estimation of Sinusoidal Signal Parameters

Yannis Pantazis, Olivier Rosec, and Yannis Stylianou

***Abstract*—While the problem of estimating the amplitudes of sinusoidal components in signals, given an estimation of their frequencies, is linear and tractable, it is biased due to the unavoidable, in practice, errors in the estimation of frequencies. These errors are of great concern for processing signals with many sinusoidal like components as is the case of speech and audio. In this letter, we suggest using a time-varying sinusoidal representation which is able to iteratively correct frequency estimation errors. Then the corresponding amplitudes are computed through Least Squares. Experiments conducted on synthetic and speech signals show the suggested model's effectiveness in correcting frequency estimation errors and robustness in additive noise conditions.**

***Index Terms*—Amplitude estimation, frequency estimation, sinusoidal modeling, time-varying models, estimation theory.**

I. Introduction

SINUSOIDAL models are widely used in signal processing for analysis, modeling and manipulation of time-series from speech and audio to radar and sonar (see [1], [2] and the references therein). In the literature, many techniques for the estimation of the sinusoidal parameters have been proposed. A simple and fast method for sinusoidal parameter estimation uses Fourier spectrum where the locations of the peaks of the spectral magnitude are the estimated frequencies and the values of the Fourier transform at these frequencies are the estimated complex amplitudes [1], [3]. Although FFT-based spectral estimation is asymptotically unbiased and efficient, it is biased for finite length data [4]. Extensions of the basic FFT method such as quadratically interpolated FFT (QIFFT) [5], [6] and reassigned spectrogram [7] have been proposed in the literature. A survey study by Keiler and Marchand [8] compares various FFT-based amplitude and frequency estimators. FFT-based methods can be considered as local since the parameters of each sinusoidal component is estimated without taking into account the possible influence of neighboring sinusoidal components. For large windows with sufficiently narrow main frequency lobe, this influence is negligible. However, using long windows the stationarity hypothesis for the analyzed signal may be violated. Consequently, the frequency estimation provided by Fourier transform-based methods is rather unreliable in nonstationary environments.

Another approach for sinusoidal parameter estimation is to use global estimation methods, through the minimization of a Least Squares (LS) criterion. For sinusoidal signals, such a criterion is highly nonlinear when the frequencies of the sinusoidal components are unknown. Hence, the estimation procedure is usually split into two steps: i) estimation of the frequencies and ii) estimation the complex amplitudes given the estimated frequencies [9], [4], [10], [11]. A major disadvantage of splitting the estimation into two subproblems is that the estimation of the complex amplitudes are severely biased when the estimation of the frequencies is not accurate. In practice, errors in frequency estimation inevitably occur when sources of interference such as noise or closely-spaced sinusoids are present.

In this paper, we consider a time-varying model for the sinusoidal parameter estimation which is immune to small frequency estimation errors. The proposed model was initially introduced by Laroche [12] for audio analysis of percussive sounds. In this model, a complex polynomial is used in order to capture fast variations within each frequency component, providing access to the instantaneous amplitudes but also—and more interestingly—to the instantaneous frequencies. Focusing on the first order model, we showed in [13] that this model is equivalent to a time-varying quasi-harmonic representation (referred to as QHM for Quasi-Harmonic Model). We showed that by proper decomposition of QHM parameters, errors in the initially estimated frequencies of sinusoidal components of the signal can be identified and then corrected. Thus, an algorithm is suggested which iteratively improves the estimated frequencies and provides unbiased amplitude estimates. The performance of the proposed estimation method is studied and boundaries of frequency errors are provided that ensure the convergence of the frequency correction algorithm. Frequency estimation experiments using short analysis windows show that the proposed method outperforms traditional FFT-based methods, especially in the case of closely-spaced sinusoids, thus underlying the ability of the suggested signal representation to perform a high resolution frequency analysis. Finally, the robustness to noise is established since the obtained estimators asymptotically reach their Cramer–Rao lower bounds even in adverse noisy conditions.

The paper is organized as follows. In Section II, the QHM model as well as its main properties are presented. Section III establishes the conditions upon which the QHM model can be used to estimate sinusoidal component. Section IV illustrates the robustness of the proposed method in additive noise, while Section V presents results on speech signals. Finally, Section VI concludes the paper.

II. Quasi-Harmonic Model

Let us consider a signal $x(t)$ consisting of K complex sinusoids:

$$x(t) = \sum_{k=1}^{K} c_k e^{j2\pi f_k t} \tag{1}$$

where f_k and c_k denote the frequency and complex amplitude, respectively, of the kth sinusoid. In order to compute the complex amplitudes through LS, we need to have estimates of frequencies $\{\hat{f}_k\}_{k=1}^{K}$ of the sinusoidal components. Such estimates

Manuscript received November 05, 2009; revised January 27, 2010. First published February 17, 2010; current version published March 26, 2010. This work was supported by Orange Labs through Industrial Contract NÂ 200141932 with FORTH. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Mark Gales.

Y. Pantazis and Y. Stylianou are with Computer Science Department, University of Crete and ICS-FORTH, Heraklion, Greece (e-mail: pantazis@csd.uoc.gr; yannis@csd.uoc.gr).

O. Rosec is with Orange Labs TECH/ASAP/VOICE, Lannion, France, (e-mail: olivier.rosec@orange-ftgroup.com).

Digital Object Identifier 10.1109/LSP.2010.2043153