European Journal of Operational Research 232 (2014) 572–583

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Stochastics and Statistics

Stochastically weighted stochastic dominance concepts with an application in capital budgeting

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ARTICLE INFO

Article history:
Received 19 October 2012
Accepted 4 August 2013
Available online 15 August 2013

Keywords:
Stochastic programming
Stochastic dominance
Risk management
Chance constraint
Integer programming

ABSTRACT

The problem of comparing random vectors arises in many applications. We propose three new concepts of *stochastically weighted dominance* for comparing random vectors X and Y. The main idea is to use a random vector V to scalarize X and Y as V^TX and V^TY, and subsequently use available concepts from stochastic dominance and stochastic optimization for comparison. For the case where the distributions of X, Y and V have finite support, we give (mixed-integer) linear inequalities that can be used for random vector comparison as well as for modeling of optimization problems where one of the random vectors depends on decisions to be optimized. Some advantages of the proposed new concepts are illustrated with the help of a capital budgeting example.

1. Introduction

Many decision situations require comparison of random vectors. Examples of such situations arise in multi-period reward in dynamic programming (Dentcheva and Ruszczyński, 2008), multi-criteria decision making (Hu and Mehrotra, 2012), risk adjusted budget allocation (Hu et al., 2011), health applications (Armbruster and Luedtke, 2010), and capital budgeting Graves et al. (2003). The concept of stochastic dominance can be used for comparing random variables and vectors (see, e.g., Shaked and Shanthikumar, 1994; Müller and Stoyan, 2002 and Levy, 2006 for comprehensive treatments of the topic). A well-known approach to compare random variables is to compare their expected utility, for a given utility function u that represents the decision maker's preferences. Stochastic dominance circumvents the problem of assuming knowledge of a decision maker's utility function by requiring that $\mathbb{E}[u(X)] \geqslant \mathbb{E}[u(Y)]$ for *all* u in a certain class $\mathfrak{U}$ of utility functions. The concept of utility-based comparison naturally generalizes to the multivariate case by using multivariate utilities. However, as observed by Zaras and Martel (1994) and Nowak (2004), it may become conservative in that setting. Moreover, testing dominance relationship in the multivariate case can be difficult, although recent work by Armbruster and Luedtke (2010) provides some new tools for that.

In this paper we propose three new concepts of *stochastically weighted dominance*. These concepts build on the idea of weighted scalarization of the random vectors. We informally present the basic ideas below to facilitate discussion on their benefits. We say that X dominates Y in the multivariate linear sense if

$$\mathbb{E}[u(v^TX)] \geqslant \mathbb{E}[u(v^TY)], \quad \forall v \in \mathfrak{V}, \ \forall u \in \mathfrak{U}. \tag{1.1}$$

A standard approach, known as multivariate linear stochastic dominance, is to use $\mathfrak{V} = \mathbb{R}^n_+$ (see, e.g., Müller and Stoyan, 2002 and Dentcheva and Ruszczyński, 2009). Homem-de-Mello and Mehrotra (2009) and Hu et al. (2012) allow $\mathfrak{V}$ to be an arbitrary polyhedral and a convex set, respectively. In our new concepts we will allow a stochastically weighted scalarization by introducing a probability measure indicating the relative importance of each vector of weights. The use of random weights was also studied in Hu and Mehrotra (2012) to develop stochastic-weight robust models for multi-stochastic objective optimization problems without the framework of stochastic dominance.

In the first concept, called stochastically weighted dominance in average (see definition in (SWD-Avg)), we require that condition (1.1) hold not for all $v \in \mathfrak{V}$ but just on the average with respect to a distribution supported on the set $\mathfrak{V}$. In the second concept, called stochastically weighted dominance with chance (see definition in (SWD-Chance)), we require that condition (1.1) hold for a given fraction of the $v \in \mathfrak{V}$ instead of all v. The third concept, called almost stochastically weighted dominance (see definition in (SWD-Almost)), can be roughly interpreted as allowing a tolerance on the right-hand side of (1.1).

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0377-2217/$ - see front matter
http://dx.doi.org/10.1016/j.ejor.2013.08.007