Mathematical modeling of heart rate and blood pressure variations due to changes in breathing pattern

Layli S. Goldoozian[1] and Edmond Zahedi[2]
School of Electrical Engineering, Sharif University of Technology
Tehran, Iran
[1]lgoldoozian@ee.sharif.edu, [2]zahedi@sharif.edu

Abstract—**Analysis of the heart rate (HR) variation due to respiration, known as respiratory sinus arrhythmia (RSA), is a method to assess the autonomic nervous system (ANS) function. In this paper a physiologically-based mathematical model consisting of the cardiorespiratory system and ANS control has been used in order to study the cardiovascular response (mean arterial blood pressure and HR) to breathing with different respiration rates and tidal volumes. Simulation results show that RSA has its maximal amplitude at the respiration frequency of 0.12 Hz and that RSA amplitude varies linearly by tidal volume. These results are in agreement with real data from the literature. In addition, the amplitude of the variations of the simulated blood pressure (BP) in each respiratory cycle is maximum at almost the same respiratory rate as RSA.**

Keywords-autonomic nervous system; cardiorespiratory system; respiratory sinus arrhythmia.

I. INTRODUCTION

The autonomic nervous system (ANS) maintains internal homeostasis by regulating cardiovascular and other vital systems [1]. The ANS does this regulation by its parasympathetic and sympathetic nerves. ANS dysfunction happens as a result of some diseases like diabetes. Although there are some methods to evaluate the ANS (like sit-to-stand, deep paced breathing and Valsalva tests), there is still the need for a method for a convenient and online assessment of its function [2].

System identification methods have been employed to model the relation between blood pressure (BP), heart rate (HR) and respiration and an index of the ANS function utilizing the autonomic related parameters of the models was defined [2], [3]. Some other researchers have modeled the autonomic control of the cardiorespiratory system using more physiologically based models [4], [5]. The cardiorespiratory system has also been modeled by electrical models where the effect of the ANS has been described by mathematical equations. These models have been used to analyze the autonomic response to changes in hydrostatic position of the body (sit-to-stand or head-up tilt tests) [4], [5]. These physiological models have also been used to simulate the response to Valsalva manoeuvre [6], [7].

One of the clinical methods to assess the ANS function is to measure the changes in heart rate due to deep paced breathing [1], as neuronal and neuromechanical mechanisms are involved in the respiratory induced changes in vital signals.

In this study, a mathematical model for the ANS control of the cardiorespiratory system has been used to study the changes in blood pressure and HR variations as a result of changes in respiration rate and tidal volume. More specifically, a lumped electrical model has been considered for the cardiorespiratory system. Also the autonomic control of the cardiorespiratory system has been mathematically formulated. Results of the simulation are finally compared with experimental results reported in the literature.

II. MODELLING

A. Cardiovascular System

The cardiovascular system is represented by a lumped-parameter model of the systemic loop (Fig. 1) consisting of the left ventricle, which pumps blood to the arteries and back to the left ventricle [8]:

$$\dot{P}(t) = \frac{P(t)}{R_c C_a} + \frac{H(t)\Delta V}{C_a} \quad (1)$$

where:

- *H(t)* is the instantaneous heart rate,
- *P(t)* is the mean arterial pressure,
- ΔV is the stroke volume.
- R_c and C_a are the total arterial resistance and compliance respectively.

For simplicity, the operation of the model is considered at the mean arterial pressure, ignoring the pulsatile component of blood pressure.

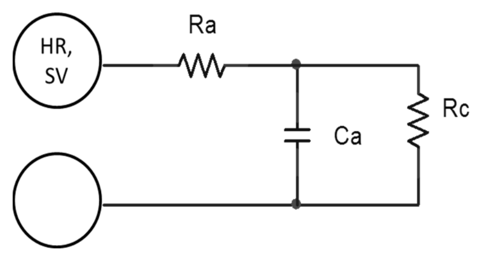

Fig. 1 Electrical lumped model of the cardiovascular system. Left and right ventricles are represented as an inflow source (upper circle) and an outflow sink (lower circle) respectively.