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# Profit and Risk Measures in Oil Production Optimization ⋆

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**Abstract:** In oil production optimization, we usually aim to maximize a deterministic scalar performance index such as the profit over the expected reservoir lifespan. However, when uncertainty in the parameters is considered, the profit results in a random variable that can assume a range of values depending on the value of the uncertain parameters. In this case, a problem reformulation is needed to properly define the optimization problem. In this paper we describe the concept of risk and we explore how to handle the risk by using appropriate risk measures. We provide a review on various risk measures reporting pro and cons for each of them. Finally, among the presented risk measures, we identify two of them as appropriate risk measures when minimizing the risk.



*Keywords:* Production Optimization, Oil production, Risk measures, Stochastic problems

## 1. INTRODUCTION

In oil production optimization, we are in general interested in maximizing an economic measure, like the profit or the net present value (NPV), over the expected reservoir life time. When uncertainty is taken into account, the profit is not a single quantity but has a probability distribution, i.e. the profit is described by a random variable $\psi$. An optimization problem involving $\psi$ in terms of a control input $u$, must express $\psi$ as a scalar quantity. Traditionally, this single quantity is the expected profit (Van Essen et al., 2009; Capolei et al., 2013). By using only the expected profit, however, we are not able to include others important indicators, that shape the profit distribution $\psi$, such as the profit deviation and the risk preference. The role of a measure of deviation is to quantify the variability of a random variable $\psi$ and the uncertainty in $\psi$ is often measured by the standard deviation of $\psi$, e.g. in classical portfolio theory (Markowitz, 1959), the standard deviation $\sigma(\psi)$ is used to quantify uncertainty in returns of financial portfolios. In the oil community, Bailey et al. (2005); Alhuthali et al. (2010); Yeten et al. (2003) propose to reduce the uncertainty in profit by including the standard deviation in the cost function. In many decision problems dealing with safety and reliability, risk is often interpreted as the probability of a dreadful event or disaster (Ditlevsen and Madsen, 1996; Rockafellar and Royset, 2010), and minimizing the probability of a highly undesirable event is known as the safety-first principle (Roy, 1952). In this paper we identify the risk as a measure of the risk of loss. When speaking of such a measure applied to the random profit, $\psi$, we have in mind that higher outcomes of $\psi$ are welcome while lower outcomes are disliked. To reduce the risk of loss then, we seek to lower the probability of the low profits. Certainly, deviation and risk are related concepts and often these terms are used interchangeably, e.g. in finance, we can interpret the profit volatility, measured by the standard deviation, as risk. Following this idea, Capolei et al. (2015) introduce the mean-variance criterion for production optimization and suggest to use the Sharpe ratio as a systematic procedure to optimally trade-off risk and return. They interpret the standard deviation as a measure of risk. However, the mean-variance approach is more suited to reduce the profit uncertainty than to reduce the risk of loss. Fig. 1 illustrates two drawbacks of the mean-variance framework when used to measure risk preferences. First of all, the mean variance approach is insensitive to the profit shape distribution. Fig.1a is a sketch representing different profit distributions having the same values for mean and the variance. In the mean-variance framework these distributions yield the same risk preference. In Fig.1b instead, the distributions in blue have a lower standard deviation, $\sigma$, than the distribution in black. If we use the standard deviation as a risk measure, the blue distributions have a lower risk than the black profit distribution, no matter what their expected values are. Furthermore, the standard deviation as a measure of risk is symmetric, which means that it penalizes higher profits and lower profits symmetrically. This last shortcoming have been recognised by Markowitz (1959) who proposed to use the semideviation instead. However, even by using the semideviation, we still do not have common properties that make sense both for a risk

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