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Constraint generation methods for robust optimization in radiation therapy

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ABSTRACT

We develop a constraint generation solution method for robust optimization problems in radiation therapy in which the problems include a large number of robust constraints. Each robust constraint must hold for any realization of an uncertain parameter within a given uncertainty set. Because the problems are large scale, the robust counterpart is computationally challenging to solve. To address this challenge, we explore different strategies of adding constraints in a constraint generation solution approach. We motivate and demonstrate our approach using robust intensity-modulated radiation therapy treatment planning for breast cancer. We use clinical data to compare the computational efficiency of our constraint generation strategies with that of directly solving the robust counterpart.

1. Introduction

Robust Optimization (RO) deals with optimization problems in which some problem parameters are uncertain and modeled as belonging to an "uncertainty set" [1,2]. One of the areas in which robust optimization has been applied is radiation therapy (RT) treatment planning. In RT, the goal is to deliver radiation beams from different angles to a cancer patient so that the beams intersect at the cancerous target (i.e., a tumor), while sparing as much of the surrounding healthy tissue as possible. Robust optimization has been used to manage uncertainties in RT treatment planning problems including uncertainties in patient geometry [3], dose calculations [4], breathing motion [5–10], and range and setup errors in proton therapy [11–14].

Much effort in RO is placed on deriving tractable robust counterparts, which are finite-sized deterministic equivalents to the original RO problem. However, the resulting robust counterpart can still be quite large and computationally challenging to solve for real-world problem instances. In radiation therapy treatment planning for example, the original problem is often of a very large scale and the robust counterpart is even larger. Therefore, there is a need for specialized solution methods to solve these problems.

Decomposition methods, which have a long history [15,16], represent a wide range of methods that can be used to solve large-scale optimization problems. Constraint generation is one type of decomposition method that has been used extensively to solve large-scale optimization problems in applications such as timetable scheduling [17], network reliability [18], network design [19], facility location [20], and network interdiction [21,22]. Oskoorouchi et al. [23] developed an interior point constraint generation algorithm for semi-infinite problems that was applied to radiation therapy.

In this paper, we develop a family of constraint generation strategies to solve large-scale robust optimization problems in radiation therapy. We focus on problems with multiple sets of robust constraints, which necessitates exploring different strategies for choosing constraints to be added at each iteration. We test several strategies for finding and adding constraints efficiently. We also compare the computational efficiency of the constraint generation methods with that of directly solving the robust counterpart. Our solution approach is motivated by the robust intensity-modulated radiation therapy (IMRT) treatment planning problem for breast cancer, in which there exists a large number of robust constraints [8].

2. Breast cancer IMRT treatment planning

Previously, a robust optimization model that incorporated conditional value-at-risk (CVaR) [24,25] was developed for breast

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