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Invited Review

Edge coloring: A natural model for sports scheduling

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ABSTRACT

In this work, we consider some basic sports scheduling problems and introduce the notions of graph theory which are needed to build adequate models. We show, in particular, how edge coloring can be used to construct schedules for sports leagues. Due to the emergence of various practical requirements, one cannot be restricted to classical schedules given by standard constructions, such as the circle method, to color the edges of complete graphs. The need of exploring the set of all possible colorings inspires the design of adequate coloring procedures. In order to explore the solution space, local search procedures are applied. The standard definitions of neighborhoods that are used in such procedures need to be extended. Graph theory provides efficient tools for describing various move types in the solution space. We show how formulations in graph theoretical terms give some insights to conceive more general move types. This leads to a series of open questions which are also presented throughout the text.

1. Introduction

Multiple agents of sports events, such as organizers, players, referees, fans, journalists, medical doctors, and airlines play an important role in professional sports leagues and tournaments. Major events such as the Olympic Games and the Football World Cup create thousands of jobs and economic opportunities to their hosts (Kendall, Knust, Ribeiro, & Urrutia, 2010; Rasmussen & Trick, 2008).

Many discrete problems in several areas can be formulated in graph theoretical terms and then solved using graph algorithms (see, e.g., Ebert, 1987). In this work, we show how basic concepts of graph theory provide a natural and very adequate tool for formulating and solving some of the problems which tournament organizers face. We will start by taking into account only some of the most frequent requirements.

We assume throughout the text that we have a round robin tournament involving an even number n of teams. Every game involves two teams, say i and j. Therefore, it is natural to associate each team with a vertex of a graph and every game involving i and j with an edge (i, j) of this graph. Fig. 1 shows an edge-colored complete graph corresponding to a tournament of $n = 4$ teams.

The primary goal of tournament scheduling is to find an assignment of each game to some time slot (or round) such that each team plays at most one game in each round and the total number of rounds is as small as possible. If each team has p games to play, any schedule clearly needs at least p rounds. For convenience, we associate each round with a specific color. Therefore, each schedule corresponds to an assignment of a color to each edge of the corresponding graph.

Games are divided into subsets F_s, each of them formed by all games assigned to a specific round s. Fig. 1 illustrates a tournament schedule with $n = 4$ teams represented by a complete graph K_4. Edges of subset F_1 are represented by dashed lines, while those of F_2 are represented by straight solid lines and edges of F_3 are represented by double solid lines. Each of these subsets represents the set of games played in a given round. We observe that, in this case, in every subset F_s of games each team is matched with exactly one other team, forming a perfect matching.

A tournament is said to be *compact* if each team plays exactly once in each round. In a very typical tournament format among n teams, each team plays exactly once (resp. twice) against all other teams in a given number of rounds. This type of competition format is called a single round robin (SRR) (resp. double round robin (DRR)) tournament. In view of simplicity, we consider the problem of scheduling a compact SRR tournament involving an even number of teams. The ideas and results presented here for SRR

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