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Application of Signal Processing to the Analysis of Financial Data

As the current recession grew to the point of being considered the worst economic downturn since the Great Depression of 1929, the news media has not only covered daily developments, but it has also discussed its causes and the lessons that can be learned. A few of the published articles, such as [1] and [2], shed some light on the fact that signal processing techniques play an important role in today's finances. Indeed, today's financial analysis and risk managers depend on mathematical tools that, at their core, are based on signal processing techniques. In this article, we highlight some of these techniques used to represent and predict the main features of price evolution and to classify stock so as to design diversified investment portfolios.

IS SIGNAL PROCESSING USEFUL IN ANALYZING FINANCIAL DATA?

Assume a manufacturer is asked by a client to sell some stock products to him at any moment in time that the client chooses within the next six months, but at today's prices rather than that moment's prices. Obviously, the client is prepared to pay today a premium to the manufacturer for the privilege of exercising this option. How much should the manufacturer ask so that the deal proves profitable for him? And how will the client choose the optimal point in time to exercise the option he paid for? Both the client and the manufacturer will have to monitor the price of the product and extrapolate its future evolution according to past available information. Viewing the price as a signal, this is a classical problem in signal processing.

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Assume now an investor wishes to build a portfolio out of several available stocks. The principle of portfolio diversification stipulates that the stocks selected to be included in the portfolio must be as uncorrelated as possible to reduce the risk due to price fluctuations (this is an application of the law of large numbers). In effect, then, each stock is viewed as a mixture of "hidden market trends," and the investor's goal is not only to determine these trends but also which stocks depend on which trends to estimate their correlations. This is a direct analogue of the canonical example in sound source separation, known as the "cocktail party problem": in a cocktail party, the signals reaching our ears are brouhahas of dozens of voices (corresponding to stock prices), and yet somehow we are able to isolate and focus on a specific speaker (corresponding to the hidden market trend).

VIEWING THE PRICE AS A SIGNAL, THIS IS A CLASSICAL PROBLEM IN SIGNAL PROCESSING.

TOOLS AND MODELS TO ANALYZE FINANCIAL DATA

A discussion of signal processing techniques in finance must necessarily begin with the Black-Scholes [7] model for price evolution. This is an excellent example of mathematically rigorous signal modeling that opened the door for the introduction of advanced quantitative techniques in finance and earned the Noble prize of economics to its inventors.

THE BLACK-SCHOLES MODEL

The most common signals in finance are prices (of goods, options, etc.), and a simple description of such a signal can be extracted from first principles and confirmed by fitting on financial data. Assume we know the price $S(0)$ at $t=0$; how does the price $S(t)$ evolve within a short time interval thereafter? The relative price increase (or decrease) between times t and $t+dt$, when dt is small, can be considered to consist of a linear term of slope r plus a random normal fluctuation of zero mean and variance proportional to dt (namely $\sigma^2 dt$, where σ is known as the volatility)

$$\frac{S(t+dt)-S(t)}{S(t)} = r\,dt + \sigma N(t)\sqrt{dt}.$$

We assume here that any two normal random variables $N(t)$ and $N(t')$ corresponding to nonoverlapping intervals $[t, t+dt]$ and $[t', t'+dt]$ are independent. At the limit when $dt \to 0$, this can be written as

$$\frac{dS(t)}{S(t)} = r\,dt + \sigma dW(t),$$

where W denotes a random walk, namely a continuous function whose increments $W(t+s)-W(t)$ are normal random variables of zero mean and variance equal to s, and such that increments over nonoverlapping intervals are independent. The framework within which such stochastic differential equations are studied is known as Itô's calculus [2], where this equation can be shown to admit the following closed form solution:

$$S(t) = S(0)e^{(r-(\sigma^2/2))\,t+\sigma W(t)}.$$

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