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Joint distribution of peaks and valleys in a stochastic process

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Abstract

General expressions and numerical results are presented pertaining to the occurrence of two local extrema of a stochastic process at prescribed time values. The extrema may be either peaks or valleys and the process may be either stationary or nonstationary. General formulas are presented for the rates of occurrence, the joint and conditional probability distributions, and the moments of the extreme values. These formulas are relatively simple multiple-integral expressions, but the integrands involve the joint probability density function for six random variables. The procedures are then applied for the special case of a stationary mean-zero Gaussian process for which the calculations are greatly simplified. Numerical results for three different spectral density functions demonstrate that conditioning on either only the existence or both the existence and the value of one peak can have a very significant effect on both the rate of occurrence and the probability distribution of a second peak.

Keywords: Conditional distribution; Extrema; Gaussian; Joint distribution; Occurrence rates; Peaks; Valleys

1. Introduction

Information about the values of the local extrema of a stochastic process is valuable in various models for estimating failure probabilities in mechanical or structural systems. There is the obvious situation regarding the occurrence of a critical level of stress or displacement by the dynamic response of a system, although this usually involves analysis of a global extreme value within a given period of time, rather than a strictly local maximum. Prediction of fatigue life, on the other hand, commonly involves estimation of the magnitude of a stress range from a local minimum (valley) to a local maximum (peak), or vice versa. Studying such a stress range, of course, requires information about the joint distribution of the valley and peak that define its end points. It should be noted that this study of local extrema at given time values is distinctly different from the study of the largest extrema within a given time interval. The latter problem has received much study, including the joint distribution of several of the largest extrema within the interval (e.g., [1]). This is in contrast to the problem studied here, which seems to have received little attention.

Rice [2] gave the probability distribution of a single peak or valley of a stochastic process, although there is sometimes some disagreement as to whether these formulas are exact or approximate. In the formulation used here these formulas are considered to give an exact description of conditional probability distributions [3]. The present work then extends this approach to derive conditional joint distributions for two peaks, two valleys, or a valley and a peak. From these results it is possible to derive conditional distributions (or conditional moments) of one extremum given information about another extremum. Results are presented both for the situation where only the existence of the second extremum is given, and for the case when both the existence and the value of the other extremum are known. Similarly, the usual form of Rice's formula for the rate of occurrence of extrema is extended to give a joint occurrence rate and a conditional rate given the existence of another extremum.

The current work focuses on the derivation of the joint and conditional distributions for a quite general stochastic process, and the specialization and simplification for the special case of a mean-zero, stationary Gaussian process. None of the results require the imposition of any Markov property, but they do require the existence of finite moment properties of the process and its first two derivatives. For the Gaussian situation, this

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