From Theory to Practice: Sub-Nyquist Sampling of Sparse Wideband Analog Signals

Moshe Mishali, *Student Member, IEEE*, and Yonina C. Eldar, *Senior Member, IEEE*

***Abstract*—Conventional sub-Nyquist sampling methods for analog signals exploit prior information about the spectral support. In this paper, we consider the challenging problem of blind sub-Nyquist sampling of multiband signals, whose unknown frequency support occupies only a small portion of a wide spectrum. Our primary design goals are efficient hardware implementation and low computational load on the supporting digital processing. We propose a system, named the modulated wideband converter, which first multiplies the analog signal by a bank of periodic waveforms. The product is then low-pass filtered and sampled uniformly at a low rate, which is orders of magnitude smaller than Nyquist. Perfect recovery from the proposed samples is achieved under certain necessary and sufficient conditions. We also develop a digital architecture, which allows either reconstruction of the analog input, or processing of any band of interest at a low rate, that is, without interpolating to the high Nyquist rate. Numerical simulations demonstrate many engineering aspects: robustness to noise and mismodeling, potential hardware simplifications, real-time performance for signals with time-varying support and stability to quantization effects. We compare our system with two previous approaches: periodic nonuniform sampling, which is bandwidth limited by existing hardware devices, and the random demodulator, which is restricted to discrete multitone signals and has a high computational load. In the broader context of Nyquist sampling, our scheme has the potential to break through the bandwidth barrier of state-of-the-art analog conversion technologies such as interleaved converters.**

***Index Terms*—Analog-to-digital conversion (ADC), compressive sampling (CS), infinite measurement vectors (IMV), multiband sampling, spectrum-blind reconstruction, sub-Nyquist sampling.**

I. INTRODUCTION

RADIO frequency (RF) technology enables the modulation of narrowband signals by high carrier frequencies. Consequently, man-made radio signals are often sparse. That is, they consist of a relatively small number of narrowband transmissions spread across a wide spectrum range. A convenient way to describe this class of signals is through a multiband model. The frequency support of a multiband signal resides within several continuous intervals spread over a wide spectrum. Fig. 1 depicts a typical communication application, the wideband receiver, in which the received signal follows the multiband model. The basic operations in such an application are conversion of the incoming signal to digital, and low-rate processing of some or all of the individual transmissions. Ultimately, the digital product is transformed back to the analog domain for further transmission.

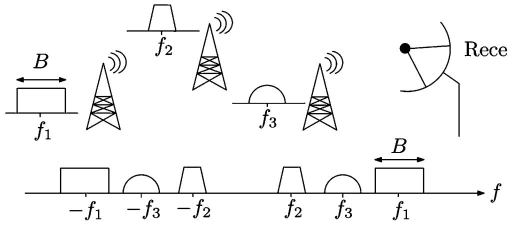

Fig. 1. Three RF transmissions with different carriers f_i. The receiver sees a multiband signal (bottom drawing).

Due to the wide spectral range of multiband signals, their Nyquist rates may exceed the specifications of the best analog-to-digital converters (ADCs) by orders of magnitude. Any attempt to acquire a multiband signal must therefore exploit its structure in an intelligent way. When the carrier frequencies are known, a common practical engineering approach is to demodulate the signal by its carrier frequency such that the spectral contents of a band of interest are centered around the origin. A low-pass filter follows in order to reject frequencies due to the other bands. Conversion to digital is then performed at a rate matching the actual information width of the band of interest. Repeating the process for each band separately results in a sampling rate which is the sum of the bandwidths. This method achieves the minimal sampling rate, as derived by Landau [1], which is equal to the actual frequency occupancy. An alternative sampling approach that does not require analog preprocessing was proposed in [2]. In this strategy, periodic nonuniform sampling is used to directly sample a multiband signal at an average rate approaching that derived by Landau. Both conventional demodulation and the method of [2] rely on knowledge of the carrier frequencies.

In scenarios in which the carrier frequencies are unknown to the receiver, or vary in time, a challenging task is to design a *spectrum-blind* receiver at a sub-Nyquist rate. In [3] and [4], a multicoset sampling strategy was developed, independent of the signal support, to acquire multiband signals at low rates. Although the sampling method is blind, in order to recover the original signal from the samples, knowledge of the frequency support is needed. Recently in [5], we proposed a fully spectrum-blind system based on multicoset sampling. Our system does not require knowledge of the frequency support in either the sampling or the recovery stages. To reconstruct the signal blindly, we developed digital algorithms that process the

Manuscript received February 22, 2009; revised October 28, 2009. Current version published March 17, 2010. Part of this work was presented at the IEEEI, 25th convention of the IEEE, Israel, December 2008. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Richard G. Baraniuk.

The authors are with the Technion—Israel Institute of Technology, Haifa 32000, Israel (e-mail: moshiko@tx.technion.ac.il; yonina@ee.technion.ac.il).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/JSTSP.2010.2042414